Exam Seat No:________________ Enrollment No:____________________

C.U.SHAH UNIVERSITY

WADHWAN CITY

University (Winter) Examination -2013

Course Name :M.Sc(Maths) Sem-I **Subject Name: -Topology-I** **Marks :70**

Duration :- 3:00 Hours **Date : 30/12/2013**

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary.
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places.
(5) Assume suitable & Perfect data if needed.

SECTION-I

Q-1

a) Define: Metric Space. (01)

b) Define open basis for a topological space. (01)

c) Which of the following are closed in standard topology? (02)
(i) $(a,\ b)$ (ii) $(a,\ b]$ (iii) $[a,\ b)$ (iv) $[a,\ b]$

d) Is $\beta = \{(a,\ b): a < b; a, b \in R\}$ a basis for a topology on R? (01)

e) Find the interior and closure of $A = (0,\ 1) \cup \{2\}$ in the standard topology on R. (02)

Q-2

a) Let $(X,\ d_1)$ and $(X,\ d_2)$ be metric spaces. (05)
For $x, y \in X$ define $d(x,\ y) = max\{d_1(x,\ y), d_2(x,\ y)\}$.
Show that d is a metric on X.

b) Let X be a non-empty set. (05)
Define $\tau_f = \{U \subset X, either\ X - U\ is\ finite\ or\ X - U = X\}$. Show that τ_f is a topology (cofinite) on X.

c) Let $(X,\ \tau)$ be a topological space and Y be a non-empty subset of X. Then prove that the collection $\tau_Y = \{V = U \cap Y : U \in \tau\}$ is a topology on Y. (04)

OR

Q-2

a) Let $(X_1,\ d_1), (X_2,\ d_2), \ldots (X_n,\ d_n)$ be metric spaces. (05)
Let $X = X_1 \times X_2 \times \ldots X_n$ and for $x = (x_1, x_2, \ldots x_n)$,
$y = (y_1, y_2, \ldots y_n) \in X$ define $d(x,\ y) = \sum_{i=1}^{n} d_i(x_i,\ y_i)$.
Show that d is a metric on X.

b) Define standard topology on R. Also show that it is a topology on R. (05)

c) Let β be a basis for a topology on X. Define $\tau = \{U \subset X : \forall x \in U, there\ exists\ B \in \beta\ such\ that\ x \in B \subset U\}$. Then show that τ is a topology on X. (04)

Q-3

a) Let Y be a subspace of X and A be a subset of Y. Then prove that (05)
$$Cl_Y(A) = Cl_X(A) \cap Y.$$

b) Let X and Y be topological spaces and $f: X \to Y$ be a function, then show that the following are equivalent. (05)
(i) f is continuous.
(ii) $f(\bar{A}) \subset \overline{f(A)}$ for every subset A of X.

c) Let X be a topological space and $A \subset X$. Then prove that $\bar{A} = A \cup A'$. (04)

OR

Q-3 a) Let X be a topological space and $A \subset X$. Then $x \in \bar{A}$ if and only if $U \cap A \neq \phi$ for every neighbourhood U of x. (05)

b) Let X, Y and Z be topological spaces. Let $f: X \to Y \times Z$ be a map defined as $f(x) = \big(f_1(x),\ f_2(x)\big)$ for all $x \in X$. Then show that the following are equivalent. (05)

(i) $f: X \to Y \times Z$ is continuous.

(ii) $f_1: X \to Y$ and $f_2: X \to Z$ both are continuous.

c) Let X be a topological space and $A \subset X$. Then prove that (04)

$$(A \cap B)^{\circ} = A^{\circ} \cap B^{\circ}.$$

SECTION-II

Q-4 a) Define complete metric space. (01)

b) Define compact space. (01)

c) Define locally compact space. (01)

d) Define disconnected topological space. (01)

e) Define locally connected space. (01)

f) State Urysohn's lemma. (02)

Q-5 a) Prove that every metric space is a T_3 space (regular space). (07)

b) Let X be a topological space. Then prove that X is a T_1 space if and only if (07)

$$\{x\} = \cap\, \{U : U \textit{ is a neighbourhood of } x, \forall x \in X\}.$$

OR

Q-5 a) Let X be a T_1 space. Then prove that X is a T_4 space if and only if given a closed set A in X and an open set U containing A, there exists an open set V containing A such that $\bar{V} \subset U$. (07)

b) Prove that a product of two T_3 spaces is a T_3 space. (07)

Q-6 a) Prove that $(B(X,\ R),\ \rho)$ is a complete metric space. (07)

b) Show that the cofinite topology $\big(X,\ \tau_f\big)$ is compact. (07)

OR

Q-6 a) Prove that every compact space is a limit point compact. (07)

b) Prove that the product of two connected spaces is connected. (07)

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